1.9 Curl

If $\vec{F}$ (x,y,z) is a differentiable vector field, then curl of $\vec{F}$ is defined as

$$\text{Curl } \vec{F} = \vec{\nabla}\times\vec{F} = \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\times\left(F_1\hat{i}+F_2\hat{j}+F_3\hat{k}\right)$$

$$\boxed{Curl\,\vec{F} = \hat{i}\left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)+\hat{j}\left(\frac{\partial F_1}{\partial y}-\frac{\partial F_3}{\partial z}\right)+\hat{k}\left(\frac{\partial F_2}{\partial y}-\frac{\partial F_1}{\partial z}\right)}$$

$$\boxed{Curl\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}}$$

General Properties:-

(i) Curl $\left(\vec{A}+\vec{B}\right)$=curl $\vec{A}$+curl $\vec{B}$

(ii) $\boxed{\text{Curl}\,Grad\,\phi = \vec{0}}$ i.e. $\vec{\nabla}\times\vec{\nabla}\phi = \vec{0}$

(iii) If $\vec{c}$ is constant vector, then curl $\vec{c} = \vec{0}$

(iv) $\vec{\nabla}\times(\phi A) = \vec{\nabla}\phi\times\vec{A}+\phi\left(\vec{\nabla}\times\vec{A}\right)$

(v) $\boxed{\text{Div}\,\text{Curl}\,\vec{A} = \vec{0}}$ i.e. $\vec{\nabla}.\left(\nabla\times\vec{A}\right) = \vec{0}$

(vi) $\vec{\nabla}\times\left(\nabla\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}$

(vii) If curl $\vec{F} = 0$ then field $\vec{F}$ is called ***irrotational field*** and line integral $\int_A^B \vec{F}.d\vec{r}$ is independent of the path joining any two points A and B. In above case, circulation $\oint \vec{F}.d\vec{r}$ zero for any closed path in that region.

(viii) If curl $\vec{F} = 0$, then three components of $\vec{F}$ are interrelated as

$$\frac{\partial F_3}{\partial y}=\frac{\partial F_2}{\partial z},\qquad \frac{\partial F_1}{\partial z}=\frac{\partial F_3}{\partial x},\quad \frac{\partial F_2}{\partial x}=\frac{\partial F_1}{\partial y}$$

(ix) If curl $\vec{F}=0$ it follows $\vec{F}=\vec{\nabla}\phi$ i.e. vector field $\vec{F}$ can be expressed as gradient of scalar field ϕ.

(x) If div $\vec{F}=0$ it follows $\vec{F}=\vec{\nabla}\times\vec{A}$ $\left[\because divcurl\,\vec{A}=0\right]$

Example1.6 A field $\vec{F}$ is given by $\vec{F}=x^2\hat{j}$ Calculate curl $\vec{F}$.

Sol.

$$Curl\,\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & x^2 & 0\end{vmatrix}$$

$$=\hat{i}\left\{\frac{\partial}{\partial y}(0)-\frac{\partial}{\partial z}\left(x^2\right)\right\}-\hat{j}\left\{\frac{\partial}{\partial x}(0)-\frac{\partial}{\partial z}(0)\right\}+\hat{k}\left\{\frac{\partial}{\partial x}\left(x^2\right)-\frac{\partial}{\partial y}(0)\right\}$$

$$=0\hat{i}+0\hat{j}+2x\hat{k}$$

Curl $\vec{F}=2x\hat{k}$

Physical Interpretation:

Value of F increases with x and $\vec{F}$ is directed along positive y direction.It is obvious from figure1.9, higher value of F is represented by larger arrow. Now we calculate the line integral $\oint\vec{F}.d\vec{r}$ along closed loop ABCDA in anticlockwise direction.

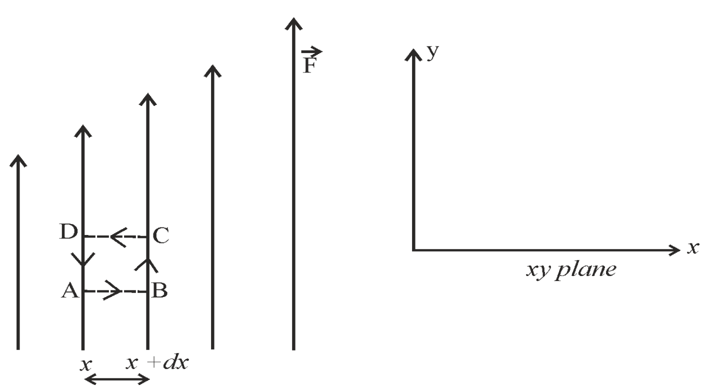

Figure 1.9

$$\oint \vec{F}.d\vec{r} = \int_{AB} \vec{F}.d\vec{r} + \int_{BC} \vec{F}.d\vec{r} + \int_{CD} \vec{F}.d\vec{r} + \int_{DA} \vec{F}.d\vec{r}$$

$$= \int_{AB} F\, dr \cos 90^0 + \int_{BC} F\, dr \cos 0^0 + \int_{CD} F\, dr \cos 90^0 + \int_{DA} F\, dr \cos 180^0$$

$$= 0 + F_{BC}(BC) + 0 + F_{DA}(-DA)$$

$$\because BC = DA = \Delta y$$

$$AB = \Delta x$$

$$\oint \vec{F} \cdot d\vec{r} = (F_{BC} - F_{DA})\Delta y$$

$$= \left[(x+\Delta x)^2 - x^2\right]\Delta y \qquad \because F = x^2$$

$$= \left[x^2 + (\Delta x)^2 + 2x\,\Delta x - x^2\right]\Delta y$$

$$= [2x + \Delta x\ ]\,\Delta x \Delta y$$

$$\frac{\oint \vec{F} \cdot d\vec{r}}{\Delta x \Delta y} = 2x + \Delta x$$

$$\frac{\oint \vec{F} \cdot d\vec{r}}{Area\ \ ABCD} = 2x + \Delta x$$

When $\Delta x \to 0,\ \Delta y \to 0$, Area ABCD become infinitesimally small and component of $curl\vec{F}$ along z direction is given by $\frac{\oint \vec{F} \cdot d\vec{r}}{Area\ \ ABCD} \cong 2x$

We know that area vector is perpendicular to plane of area. Here, we have calculated in $\oint \vec{F} \cdot d\vec{r}$ in xy plane and for anti-clockwise rotation, outward unit vector is $\vec{k}$ which is perpendicular to xy plane.

Thus $curl\vec{F}$ has component $2x$ along $\hat{k}$ direction. Similarly we can show that $curl\vec{F}$ do not have components along $\hat{i}$ and $\hat{j}$ directions.

Example1.7 If electrostatic field $\vec{E} = \frac{a\vec{r}}{r^3}$,then find curl $\vec{E}$ where *a* is constant

Sol. $\vec{E} = a\left(\dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{r^3}\right) = \dfrac{ax}{r^3}\hat{i} + \dfrac{ay}{r^3}\hat{j} + \dfrac{az}{r^3}\hat{k}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{ax}{r^3} & \dfrac{ay}{r^3} & \dfrac{az}{r^3} \end{vmatrix}$$

$$= \hat{i}\left\{\frac{\partial}{\partial y}\left(\frac{az}{r^3}\right) - \frac{\partial}{\partial z}\left(\frac{ay}{r^3}\right)\right\} - \hat{j}\left\{\frac{\partial}{\partial x}\left(\frac{az}{r^3}\right) - \frac{\partial}{\partial z}\left(\frac{ax}{r^3}\right)\right\} + \hat{k}\left\{\frac{\partial}{\partial x}\left(\frac{ay}{r^3}\right) - \frac{\partial}{\partial z}\left(\frac{ax}{r^3}\right)\right\}$$

$$\vec{\nabla} \times \vec{E} = \hat{i}\left\{az\left(-3r^{-4}\right)\frac{\partial r}{\partial y} - ay\left(-3r^{-4}\right)\frac{\partial r}{\partial z}\right\} - \hat{j}\left\{az\left(-3r^{-4}\right)\frac{\partial r}{\partial x} - ax\left(-3r^{-4}\right)\frac{\partial r}{\partial z}\right\}$$

$$+\hat{k}\left\{ay\left(-3r^{-4}\right)\frac{\partial r}{\partial x} - ax\left(-3r^{-4}\right)\frac{\partial r}{\partial y}\right\}$$

We know that $r^2 = x^2 + y^2 + z^2$ partial differential $w.r.to\ x$ gives

$$2r\frac{\partial r}{\partial x} = 2x \quad \Rightarrow \quad \frac{\partial r}{\partial x} = \frac{r}{x}$$

Similarly $\dfrac{\partial r}{\partial y} = \dfrac{y}{r}$, $\dfrac{\partial r}{\partial z} = \dfrac{z}{r}$ putting these values, $\vec{\nabla} \times \vec{E}$ becomes

$$\vec{\nabla} \times \vec{E} = \hat{i}\left\{az\left(-3r^{-4}\right)\frac{y}{r} - ay\left(-3r^{-4}\right)\frac{z}{r}\right\} - \hat{j}\left\{az\left(-3r^{-4}\right)\frac{x}{r} - ax\left(-3r^{-4}\right)\frac{z}{r}\right\}$$

$$+\hat{k}\left\{ay\left(-3r^{-4}\right)\frac{x}{r} - ax\left(-3r^{-4}\right)\frac{y}{r}\right\}$$

$$= \hat{i}(0) + \hat{j}(0) + \hat{k}(0) = \vec{0}$$

Note: Here $\vec{E} = \dfrac{a\vec{r}}{r^3} = \dfrac{a}{r^2}\hat{r} = f(r)\hat{r}$ where $f(r) = \dfrac{a}{r^2}$ is function of radial distance. $\vec{\nabla} \times \vec{E} = 0 = \vec{\nabla} \times f(r)\hat{r}$

Similarly we can prove that any vector field which can be written as $f(r)\hat{r}$,curl of that field will be zero. That type of field is known as central field. So curl of central field is always zero and it is conservative in nature. Electrostatic field and gravitational field are central fields.

Example1.8 A vector field is given by $\vec{F} = f(x)\,\hat{i} + f(y)\,\hat{j} + f(z)\hat{k}$, then prove that curl $\vec{F} = 0$, where $f(x)$ is function of x only, $f(y)$ is function of y only, $f(z)$ function of z only.

Sol.

$$Curl\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x) & f(y) & f(z) \end{vmatrix}$$

$$= \hat{i}\left\{\frac{\partial}{\partial y} f(z) - \frac{\partial}{\partial z} f(y)\right\} - \hat{j}\left\{\frac{\partial}{\partial x} f(z) - \frac{\partial}{\partial z} f(x)\right\} + \hat{k}\left\{\frac{\partial}{\partial x} f(y) - \frac{\partial}{\partial y} f(x)\right\}$$

$$= \hat{i}\{0-0\} - \hat{j}\{0-0\} + \hat{k}\{0-0\} = \vec{0}$$

1.10 Self Learning Exercise-II

Very Short Answer Type Questions

Q.1 If $\vec{F} = (x^2+1)\hat{i} + 3y^3\hat{j} + z^{1/2}\hat{k}$ then find curl $\vec{F}$

Q.2 For particular path $\oint \vec{F}.d\vec{r} = 0$ .Does it imply $\vec{\nabla} \times \vec{F} = 0$

Short Answer Type Questions

Q.3 A vector field is $\vec{F} = \hat{i} \times \vec{r}$. Is this field solenoidal?

Q.4 Continuity equation is given by $\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$,where $\vec{J} = ax\hat{i} + by\hat{j}$ then find ρ where *a* and *b* are constants.

1.11 Summary

(i) Scalar Triple Product $\vec{A}.(\vec{B} \times \vec{C})$

$$\vec{A}.\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

(ii) Vector Triple Product $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}.\vec{C}\right)-\vec{C}\left(\vec{A}.\vec{B}\right)$$

(iii) $grad\phi=\vec{\nabla}\phi=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\phi$

(iv) $div\vec{F}=\vec{\nabla}\,.\,\vec{F}=\left(\frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z}\right)$ Where $\vec{F}=F_1\hat{i}+F_2\hat{j}+F_3\hat{k}$

(v) Curl $\vec{F}=\vec{\nabla}\times\vec{F}=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\times\left(F_1\hat{i}+F_2\hat{j}+F_3\hat{k}\right)$

$$Curl\,\vec{F}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

1.12 Glossary

Collinear : Lying in the same straight line

Equipotential surfaces: All points on an equipotential surface have the same potential.

Ellipsoid : a geometric surface described by standard equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ where $\pm a$, $\pm b$, $\pm c$ are the intercepts on the *x, y,* and *z* axes.

1.13 Answer to Self Learning Exercises

Answer to Self Learning Exercise-I

Ans.1: $\cos^{-1}\frac{4}{\sqrt{41}}$

Ans.2: Let $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = \hat{i} + \hat{j}$, $\vec{C} = \hat{j} + \hat{k}$

Ans.3: $\left|\vec{a}_1 \cdot \left(\vec{a}_2 \times \vec{a}_3\right)\right| = \dfrac{a^3}{2}$

Ans.4: $2\vec{r}$

Answer to Self Learning Exercise-II

Ans.1: $Crul\ \vec{F} = \vec{0}$ since $Crul\ \vec{F} = f(x)\hat{i} + f(y)\hat{j} + f(z)\hat{k}$

Ans.2: No, if $\oint \vec{F}.d\vec{r} = 0$ for all paths then $\vec{\nabla} \times \vec{F} = 0$

Ans.3: Yes, because $\nabla.\vec{F} = 0$

Ans.4: $\rho_0 - (a+b)t$ where ρ_0 is constant

1.14 Exercise

Section A : Very Short Answer Type Questions

Q.1 Diamond unit cell consists a basis in which one atom at origin 'O' and another atom at point P $\left(\dfrac{a}{4}, \dfrac{a}{4}, \dfrac{a}{4}\right)$. Find the angle made by $\vec{OP}$ with x, y, z axes.

Q.2 $\vec{F} = 2r\hat{r} + \dfrac{3a\vec{r}}{r^2}$,Is this field $\vec{F}$ irrotational?

Q.3 If $\vec{A} = x^2\hat{i} + y^2\hat{j}$,then find $Div\,\vec{A}$

SectionB : Short Answer Type Questions

Q.4 A particle is displaced from position $\vec{r}_1 = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ to $\vec{r}_2 = \left(4\hat{i} + 2\hat{j} + \hat{k}\right)$ under a constant force $\vec{F} = \left(\hat{i} + \hat{j} + \hat{k}\right)$ .All units are in S.I. Calculate the work done by the force on the particle for given displacement.

Q.5 Reciprocal lattice vector of a unit cell are $\dfrac{2\pi}{a}\hat{i}$, $\dfrac{2\pi}{b}\hat{j}$, $\dfrac{2\pi}{c}\hat{k}$. Find the volume of the cell formed by these reciprocal lattice vectors.

Q.6 A solenoidal vector field is given by $\vec{F} = x\hat{i} - by\hat{j} + cz\hat{k}$. Find the relation between c and b.

Q.7 Position vector of a moving particle is given by $\vec{r} = \left(t\hat{i} + t^2\hat{j}\right)$. Find the areal velocity of the particle about origin at time t=2 sec. All units are in SI and areal velocity is $\frac{d\vec{A}}{dt} = \frac{1}{2}\left(\vec{r} \times \vec{v}\right)$

Section C : Long Answer Type Questions

Q.8 A particle of mass m is moving with velocity $\vec{v} = \vec{w} \times \vec{r}$, where $\vec{w}$ is constant vector. Angular momentum of the particle is $\vec{L} = m\,\vec{r} \times \left(\vec{w} \times \vec{r}\right)$ then find curl $\vec{L}$.

Q.9 Find curl of the following vector fields-

(i) $\vec{F}_1 = y\,\hat{i}$ (ii) $\vec{F}_2 = x\,\hat{j}$ (iii) $\vec{F}_3 = \left(\vec{F}_1 + \vec{F}_2\right) = y\hat{i} + x\hat{j}$

Q.10 A wire of radius 'R' carries current along positive z direction. Magnetic field inside the wire is $\vec{B} = \frac{\mu_0}{2}\left(\vec{J} \times \vec{r}\right)$ where $\vec{J} = J\vec{k}$ is uniform current density. Calculate the curl $\vec{B}$ inside the wire.

Q.11 If $\vec{v} = \vec{\omega} \times \vec{r}$ then find $\vec{\nabla}.\vec{v}$ and $\vec{\nabla}|\vec{v}|$ where $\vec{\omega} = \omega\hat{k}$

1.15 Answers to Exercise

Ans.1: $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ with each axis

Ans.2: Yes ,because $\vec{F} = f(r)\hat{r}$

Ans.3: $2x + 2y$

Ans.4: $\vec{W} = \hat{F}.\left(\vec{r}_2 - \vec{r}_1\right) = -2\ Joule$

Ans.5: $\dfrac{(2\pi)^3}{abc}$

Ans.6: $\vec{\nabla}.\vec{F}=0$ gives $b-c=1$

Ans.7: Hint $\vec{v}=\dfrac{d\vec{r}}{dt}$, $\dfrac{d\vec{A}}{dt}=2\hat{k}$

Ans.8: $-3m\vec{v}$

Ans.9: (i) $curl\ \vec{F}_1=-\hat{k}$ (ii) $curl\ \vec{F}_2=\hat{k}$ (iii) $curl\ \left(\vec{F}_1+\vec{F}_2\right)=0$

Note that all these three fields $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are not in the form of $\left[f(x)\hat{i}+f(y)\hat{j}+f(z)\hat{k}\right], curl\,\vec{F}_1\neq 0$, $curl\,\vec{F}_2\neq 0, curl\,\vec{F}_3=0$ .Thus field which is not in the form of $\left[f(x)\hat{i}+f(y)\hat{j}+f(z)\hat{k}\right]$ its curl may be zero or may not be zero.

Ans.10: $Curl\,\vec{B}=u_0\,\vec{J}$

Ans.11: $\vec{\nabla}.\vec{v}=0$, $\vec{\nabla}\left|\vec{v}\right|=\dfrac{x\hat{i}+y\hat{k}}{\sqrt{x^2+y^2}}$

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UNIT- 2
Coordinate Systems

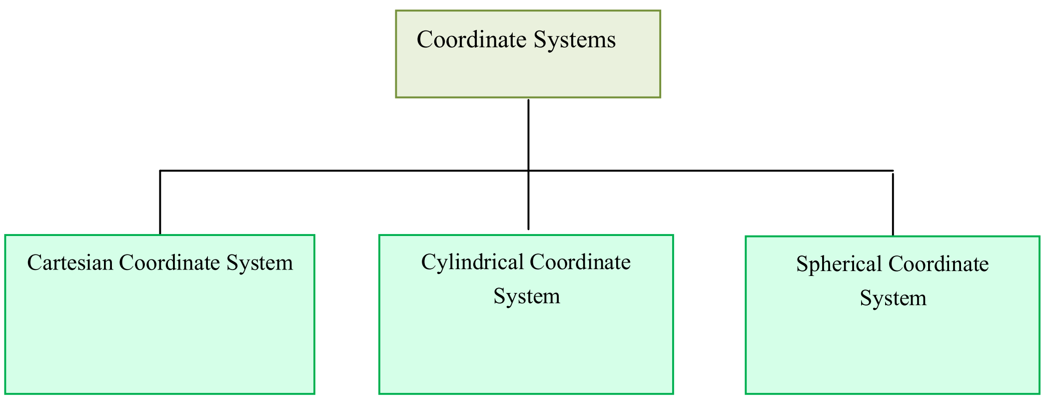
Structure of the Unit

2.0 Objectives

The chapter provides a simple formalism for expressing certain basic ideas about the coordinates system. The aim of this chapter is to enable the reader to understand the relationship and formalisms between different coordinates system. Topics have covered the algebra and differential calculus of various operations. Transformation between coordinates system is also well explained. Added feature on curvilinear coordinates system are extremely useful in the study of this chapter.

2.1 Introduction

Coordinate system is a basic idea which is used to define the position of an object in given space. The position of an object is identified by coordinates of its concerned coordinate system. These coordinates are based on measurements of position (displacements, directions, projections, angle etc.) from a given location. There are mainly three types of coordinate systems.



We are quite familiar with Cartesian coordinate system. For systems exhibiting cylindrical or spherical symmetry, it is easy to use the cylindrical and spherical coordinate systems respectively.

2.2 Cartesian or Rectangular Coordinate System

The idea of Cartesian coordinate system was invented by (and is named after) French philosopher, physicist, physiologist, and mathematician René Descartes in the 17^{th} century. A Cartesian or Rectangular co- ordinate system usually consists of three mutually intersecting perpendicular co – ordinate axes are set – up which are labeled as X, Y and Z axes. The point where three axis (X,Y,Z) cross or intersect is called the origin (0,0,0).

In rectangular coordinates system a point P is identified by coordinates x_i, y_i, and z_i (three dimensions) where these values are all measured from the origin (see figure.1).

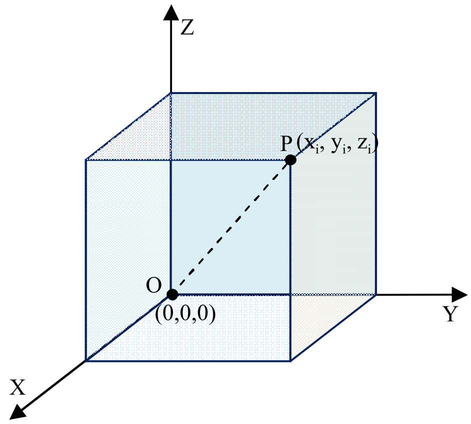


Fig.1: Location of point P in Cartesian

In a three dimensional space a point P can be identified as the intersection of three surfaces as shown in fig. When the surfaces intersect perpendicularly we have an orthogonal coordinate system. The point P (x_i, y_i, z_i) is located at the intersection of three constant surfaces i.e.,

	Ranges of Variables
x = const. (Planer Surface)	$(-\infty \le z < \infty)$
y = const. (Planer Surface)	$(-\infty \le z < \infty)$
z = const. (Planer Surface)	$(-\infty \le z < \infty)$

Unit vectors $\hat{a}_x$, $\hat{a}_y$ and $\hat{a}_z$ are perpendicular to the planer surfaces.

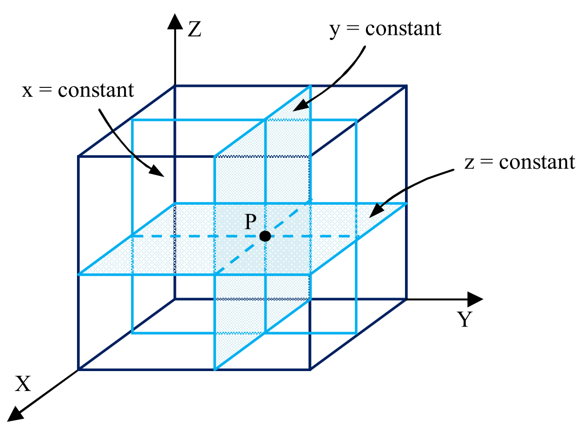


Fig.2: Location of point P in Cartesian coordinate constant surfaces

2.2.1 Differential Elements in Cartesian Coordinates

In Mathematical physics, there are three differential elements corresponding to length (l), area (A) and volume (V). These three differential elements provide an integrating function to different coordinating system.

The definition of the proper differential elements of length (*dl* for line integrals) and area (*ds* for surface integrals) can be characterized directly from the definition of the differential volume or parallelepiped (*dv* for volume integrals) in a particular coordinate system respectively.

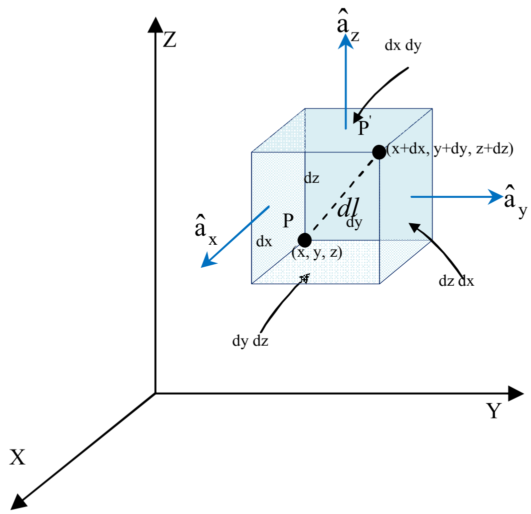


Fig.3: Constants surfaces and unit vectors in Cartesian coordinates system

In order to determine the differential element (parallelepiped) corresponding to length, area and volume in Cartesian coordinate system, let us consider a point P at the location (x,y,z). If now each coordinate's value is increased by a differential amount by (x+dx, y+dy, z+dz) as shown in fig.3, then

(1)Differential Length Element

$$\boxed{d\vec{l} = \hat{a}_x dx + \hat{a}_y dy + \hat{a}_z dz}$$

The distance between two points P and P' (diagonal length).

$$\boxed{dl = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}}$$

(2) Differential surface area element

$$\boxed{\begin{aligned} d\vec{s}_x &= dy\,dz\,\hat{a}_x \\ d\vec{s}_y &= dx\,dz\,\hat{a}_y \\ d\vec{s}_z &= dx\,dy\,\hat{a}_z \end{aligned}} \qquad \text{or} \qquad \begin{aligned} ds_x &= dy\,dz \\ ds_y &= dx\,dz \\ ds_z &= dx\,dy \end{aligned}$$

(3) Differential volume element $\boxed{dv = dx\,dy\,dz}$

2.3 Cylindrical Coordinate System

In cylindrical coordinates system a point P is specified by three coordinates (ρ, φ , z), where ρ represents a radial distance, φ an angular displacement (Azimuth Angle) and z an axial displacement (see figure.4).

In circular cylindrical coordinate system a point P can also be identified as intersection of three mutually perpendicular surfaces as follows:

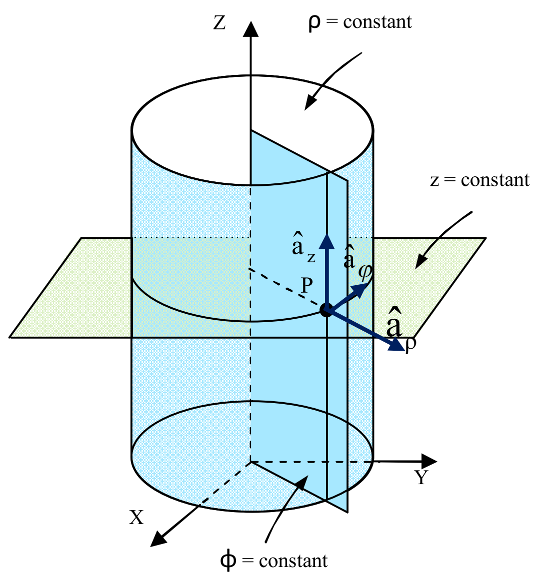


Fig.4: Constants surfaces and unit vectors in cylindrical coordinates system

	Ranges of Variables
ρ = const. (A circular cylinder);	$(0 \le \rho < \infty)$
φ = const. (A Plane);	$(0 \le \varphi \le 2\pi)$
z = const. (Another plane).	$(-\infty \le z < \infty)$

Unit vectors in cylindrical coordinates system are characterized as follow:

$\hat{a}_\rho \perp \rho = \text{const.}$ (Perpendicular to the circular cylindrical Surface)

$\hat{a}_\phi \perp \phi = \text{const.}$(Perpendicular to the planer Surface)

$\hat{a}_z \perp z = \text{const.}$(Perpendicular to the planer Surface)

2.3.1 Differential Elements in Cylindrical Coordinate System

The differential element corresponding to length, area and volume in cylindrical coordinate system can be found as shown in fig below;

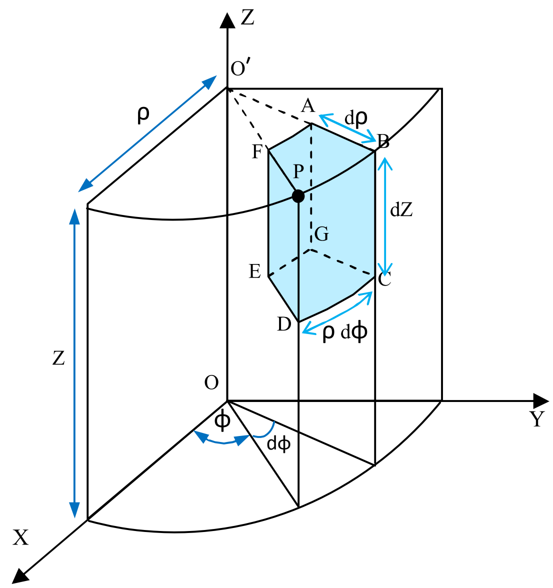


Fig.5: Differential element in cylindrical coordinates system

(1)Differential Length Element

$$d\vec{l} = \hat{a}_\rho d\rho + \hat{a}_\varphi r\, d\varphi + \hat{a}_z dz$$

(2) Differential surface area element

$$d\vec{s}_r = \rho\, d\varphi\, dz\, \hat{a}_\rho \qquad\qquad ds_r = \rho\, d\varphi\, dz$$

$$d\vec{s}_\varphi = d\rho\, dz\, \hat{a}_\varphi \qquad \text{or} \qquad ds_\varphi = d\rho\, dz$$

$$d\vec{s}_z = \rho\, d\rho\, d\varphi\, \hat{a}_z \qquad\qquad ds_z = \rho\, d\rho\, d\varphi$$

(3) Differential volume element

$$dv = \rho\, d\rho\, d\varphi\, dz$$