(ii) Highly reliable and fa...

(iii) Digital circuits can incorporate memory and ... calculators, watches, computers etc.

(iv) Digital circuits are quite insensitive to the changes in the characteristics of components, temperature fluctuations, ageing of the components, noise etc.

17.2 BINARY NUMBERS

In the *decimal system*, ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are required to express a number. The *base* or *radix* of a number system is the total number of digits used in the system. Clearly, the decimal system has the base 10 (ten). In the decimal system, the individual digits of the number represent the coefficients of some power of 10 (ten). For example, the number 1437 (one thousand four hundred thirty seven) is expanded in powers of 10 (ten) as follows:

$$1437 = 1 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 7 \times 10^0.$$

The digit at the extreme right, *i.e.* 7 is the coefficient of 10^0, the next digit from the extreme right, *i.e.* 3 is the coefficient of 10^1, and so on. Thus, from the extreme right to the left, the digits in the number are the coefficients of ascending power of 10, starting with the zeroth power.

In the *binary system*, a number is expressed by two digits, 0 and 1. The base of the binary system is thus two. In this system, the individual digits, 0 and 1, represent the coefficients of powers of 2 (two). For example, the decimal number 6 is written as

$$6 = 4 + 2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 .$$

The binary representation of the decimal number 6 is thus 110 (read as one-one-zero, not one hundred ten). Similarly, the binary equivalent of the decimal number 17 is 10001 (read as one-zero-zero-zero-one), that of the number 21 is 10101 (read as one-zero-one-zero-one) and so on. Table 17.1 shows the binary equivalents of a few decimal numbers. To avoid any possible confusion with decimal numbers, binary numbers are sometimes embraced by brackets with the base 2 written as a subscript. Thus the binary number 10101 is written as $(10101)_2$. The brackets are optional: the number can also be written as 10101_2.

A *binary digit* (1 or 0) is known as a *bit*. A group of bits with a significance is called a (or *byte*), *word*, or *code*. Usually, 'byte' represents a group of eight bits: the word 'byte' is derived from 'by eight'. For instance, the binary number 10010101 has eight digits or eight bits. The number itself is a byte. A group of four bits makes a *nibble*. Thus 1101 is a nibble.

Table 17.1 Decimal numbers and their binary equivalents

Decimal number	Binary number	Decimal number	Binary number
0	0	7	111
1	1	8	1000
2	10	9	1001
3	11	10	1010
4	100	11	1011
5	101	19	10011
6	110	26	11010

17.3 DECIMAL-TO-BINARY CONVERSION

To convert a decimal number into its binary equivalent, the decimal number is expressed as a sum of ascending power of 2. The successive coefficients of the powers of 2 represent the number in the binary system. Thus, to convert the number 7 to its binary form, we write

$$7 = 4 + 2 + 1 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 .$$

The coefficients of 2^2, 2^1, and 2^0 are 1,1 and 1, respectively. Hence the binary representation of 7 is 111 (one-one-one). Similarly, the representation of 23 is 10111, since

$$23 = 16 + 4 + 2 + 1 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

An alternative method of converting from the decimal to the binary system is to divide the decimal number progressively by 2 till the quotient is zero. The remainders of the successive divisions, written in the reverse order, give the binary number. For example, to convert the decimal 23, we proceed as follows:

$$\begin{aligned}
23 \div 2 &= 11 \ ; \quad \text{remainder} = 1\\
11 \div 2 &= 5 \ ; \quad \text{remainder} = 1\\
5 \div 2 &= 2 \ ; \quad \text{remainder} = 1\\
2 \div 2 &= 1 \ ; \quad \text{remainder} = 0\\
1 \div 2 &= 0 \ ; \quad \text{remainder} = 1 \uparrow
\end{aligned}$$

Arranging the remainders in reverse order, i.e. from bottom, up, the binary equivalent of 23 is found to be 10111.

When the decimal number is a fraction, the number is multiplied repeatedly by 2, and the carry in the integer position is recorded each time. The process is continued until the fractional part is zero, or sufficient bits of the binary equivalent have been obtained. The carries, written in the forward order, give the binary equivalent of the fraction. For example, the binary equivalent of the fraction 0.75 is determined as follows:

$$\begin{aligned}
0.75 \times 2 &= 1.50 = 0.50 \ ; \quad \text{carry} = 1\\
0.50 \times 2 &= 1.0 = 0 \ ; \quad \text{carry} = 1 \downarrow
\end{aligned}$$

The equivalent binary fraction is thus 0.11. The point used in the binary fraction is a binary point whereas that used in the decimal fraction is the decimal point. Similarly, the binary fraction equivalent to 0.55 is obtained in the following manner:

$$\begin{aligned}
0.55 \times 2 &= 1.10 = 0.10 \ ; \text{carry} = 1\\
0.10 \times 2 &= 0.20 = 0.20 \ ; \text{carry} = 0\\
0.20 \times 2 &= 0.40 = 0.40 \ ; \text{carry} = 0\\
0.40 \times 2 &= 0.80 = 0.80 \ ; \text{carry} = 0\\
0.80 \times 2 &= 1.60 = 0.60 \ ; \text{carry} = 1\\
0.60 \times 2 &= 1.20 = 0.20 \ ; \text{carry} = 1 \downarrow
\end{aligned}$$

In the last step, we get back 0.20. To obtain an approximate result, we can terminate the process at this stage, thus the approximate binary representation of the fraction 0.55 in terms of six binary digits is 0.100011. If a greater accuracy is required, the number of digits in the binary representation can be increased by continuing the multiplication by 2.

If the decimal number contains an integer part and a fraction, the two parts are separated to find the binary equivalent. For example, the number 23.55 is separated into the integer 23 and the fraction 0.55. The binary equivalent of each part is obtained separately in the manner described above. The binary representation of 23.55 is thus 10111.100011. Note that the representation is approximate since the fraction 0.55 is approximated by its six-digit representation.

The binary equivalent of a negative decimal number is obtained by putting a 'minus' sign before the binary equivalent of the corresponding positive decimal number. Thus the binary representation of 19 is 10011 and that of – 19 is – 10011.

17.4 BINARY-TO-DECIMAL CONVERSION

A binary number can be converted into its decimal equivalent by noting that the successive digits from the extreme right of a binary number are the coefficients of ascending powers of 2 beginning with the zeroth power of 2 at the extreme right. For example, the binary number 10110 is written as

$$\begin{aligned} 10110 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 16 + 0 + 4 + 2 + 0 = 22 \text{ (twenty two)}. \end{aligned}$$

Thus the decimal equivalent of the binary number 10110 is 22. Similarly,

$$\begin{aligned} 101 &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 0 + 1 = 5 \\ 1111 &= 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 4 + 2 + 1 = 15 \\ 10101 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 16 + 0 + 4 + 0 + 1 = 21 \end{aligned}$$

So, the decimal numbers equivalent to the binary numbers 101, 1111, and 10101 are 5, 15 and 21, respectively.

The decimal equivalent of a binary fraction is found by multiplying each digit in the fraction successively by 2^{-1}, 2^{-2}, 2^{-3}, ... etc., beginning with the first digit after the binary point. The products are then added to get the equivalent decimal fraction. As an example, the decimal equivalent of the binary fraction 0.1101 is

$$1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} = \frac{1}{2} + \frac{1}{4} + 0 + \frac{1}{16} = 0.5 + 0.25 + 0.0625 = 0.8125$$

Proceeding in the same manner, the decimal equivalents of 0.111 and 0.1001 are found to be 0.875 and 0.5625, respectively.

When the binary number consists of an integer part and a fraction, the decimal equivalent of each part is determined separately and then combined to obtain the decimal equivalent of the given number. For example, the decimal equivalent of 1011.111 is 11.875, since the integer part 1011 has the decimal equivalent 11 and the fraction 0.111 has the decimal equivalent 0.875.

17.5 BINARY ADDITION

The binary addition is performed with the help of the following four rules :

(*a*) 0 + 0 = 0, which implies that nothing added to nothing yields nothing.

(*b*) 1 + 0 = 1, implying that one added to nothing gives one.

(*c*) 0 + 1 = 1, implying that nothing added to one yields one.

(*d*) 1 + 1 = 10 (one-zero), implying that one added to one gives 2, the decimal equivalent of 10 (one-zero) being 2.

Some binary additions are carried out below. The numbers within brackets give the decimal equivalent.